

# Maths for Computing

## Tutorial 3

1. Show that the following arguments are valid. (Write all the steps with reasons.)
- a) Premises:  $p \rightarrow q$ ,  $(q \vee r) \wedge (\neg(q \wedge r))$ . Conclusion:  $\neg q \rightarrow (\neg p \wedge r)$ .
- b) Premises:  $\forall x(P(x) \vee Q(x))$ ,  $\forall x(\neg Q(x) \vee S(x))$ ,  $\forall x(R(x) \rightarrow \neg S(x))$ , and  $\exists x \neg P(x)$   
Conclusion:  $\exists x \neg R(x)$ . (Domain for all quantifiers are the same.)

2. Find the flaw in the below proof that shows that if  $\exists x P(x) \wedge \exists x Q(x)$  is true, then  $\exists x(P(x) \wedge Q(x))$  is true.

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|---|----------------------------------|
| 1. $\exists x P(x) \wedge \exists x Q(x)$ | Premise                          |
| 2. $\exists x P(x)$                       | Simplification of 1              |
| 3. $\exists x Q(x)$                       | Simplification of 1              |
| 4. $P(c)$                                 | Existential instantiation from 2 |
| 5. $Q(c)$                                 | Existential instantiation from 3 |
| 6. $P(c) \wedge Q(c)$                     | Conjunction from 4 and 5         |
| 7. $\exists x(P(x) \wedge Q(x))$          | Existential Generalization       |

3. Suppose an argument  $A$  has premises  $p$ ,  $q$ , and  $r$ , where each premise can be a compound proposition, and conclusion  $s$ , where  $s$  can be a compound proposition too. Show that the argument  $A$  is valid if and only if  $(p \wedge q \wedge r) \rightarrow s$  is a tautology.

4. Prove the following:
- a) If  $n$  is an integer and  $3n + 2$  is odd, then  $n$  is odd.
- b) If  $x$  is irrational, then  $1/x$  is irrational.